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# CERTIFIED PUBLIC ACCOUNT

# FOUNDATION LEVEL 1 EXAMINATION

# <u>F1.1: BUSINESS MATHEMATICS AND QUANTITATIVE METHODS</u>

## DAY: THURSDAY, 01 DECEMBER 2022

## MARKING GUIDE AND MODEL ANSWER

## QUESTION ONE

**Marking guide**

| | **criteria** | **Marks** |
|---|---|---|
| **a. i** | Award 0.5 Marks for the Mean and standard deviation formula | **1** |
| | Award 0.5 Marks for the correct answer of Mean and standard deviation | **1** |
| **a. ii** | Award 0.5 Marks for the Mean and variance formula | **1** |
| | Award 0.5 Marks for the correct answer of variance | **0.5** |
| | Award 1 Mark for the correct answer of $\sum y^2$ | **1** |
| | Award 0.5 Marks for the correct answer of Total number of errors | **0.5** |
| | Award 1 Mark for the correct answer of combined mean | **2** |
| | Award 1 Mark for the correct answer of combined variance | **1** |
| | Award 1 Mark for the correct answer of combined standard deviation | **2** |
| **Sub-total A** | | **10** |
| **b.** | Award 0.5 Marks for the Q1, Median (Q2) and Q3 formula | **1.5** |
| | Award 0.5 Marks for the correct answer of Q1, Median (Q2) and Q3 | **1.5** |
| | Award 1 Mark for the correct answer of Quartile coefficient of skewness | **1** |
| **c. i** | Award 1 Mark for mode of grouped data formula | **1** |
| | Award 1 Mark for the correct answer of mode of grouped data | **1** |
| **c. ii** | Award 1 Mark for the correct answer of mean of grouped data | **1** |
| | Award 1 Mark for the correct answer of standard deviation of grouped data | **1** |
| | Award 1 Mark for the correct total of $f_i * x_i^2$ of grouped date | **1** |
| **c. iii** | Award 1 Mark for the correct answer of the coefficient of variation of grouped data | **1** |
| **Sub-total B** | | **10** |
| Total | | **20** |

**Model Answer**

**Solution A**

**i)** Given: $\sum x = 920$ , $\sum x^2 = 5032$ $n = 200$

$Mean = \bar{x} = \frac{\sum x}{n}$

$= \frac{920}{200} = 4.6$

$Standard\ deviation = \sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$

$= \sqrt{\frac{5032}{200} - (4.6)^2} = \sqrt{4} = 2$

**ii)** Let assume that the error on the further 50 pages is y

$Mean = \bar{y} = \frac{\sum y}{n}$

$\sum y = \bar{y} * n = 4.4 * n = 4.4 * 50 = 220$

$\sigma = 2.2$

$\Rightarrow (2.2)^2 = Variance = \frac{\sum y^2}{n} - \bar{y}^2$

$(2.2)^2 = \frac{\sum y^2}{50} - (4.4)^2$

$\sum y^2 = 50 * ((2.2)^2 + (4.4)^2) = 1210$

Combining the two sets of 250 pages,

Total number of errors = $\sum x + \sum y = 920 + 220 = 1140$

Let $\bar{x}_{12}$ be the combined mean and $\sigma_{12}$ combined standard deviation

$Variance = \sigma_{(12)}^2 = \frac{\sum x^2 + \sum y^2}{n_1 + n_2} - \bar{x}_{12}^2$

$= \frac{5032+1210}{250} - (4.56)^2 = 4.1744$

Therefore, the standard deviation is $\sigma_{12} = \sqrt{\sigma_{12}} = \sqrt{4.1744} = 2.04$

**Solution B**

Let $Q_1$ and $Q_3$ be quartile 1 and quartile 3 respectively

**i)** Number of observation n is odd

Therefore, $Q_1 = \left(\frac{1}{4} X_{n+1}\right)^{th}$ $value = X^{th}_{\frac{31}{4}+1}$

$Q_3 = \left(\frac{3}{4} X_{n+1}\right)^{th}$ $value = X^{th}_{\frac{31}{4}+1} = X^{th}_{24}$

Arranging the results in order:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 23 | 26 | 29 | 30 | 31 | 31 | 37 | 37 | 38 | 38 | 41 |
| 44 | 44 | 46 | 46 | 48 | 49 | 49 | 52 | 52 | 56 | 58 |
| 59 | 61 | 61 | 61 | 63 | 70 | 75 | 76 | 85 | | |

$Q_1 = X_8^{th} = 8^{th} value = 37$

$Q_3 = X_{24}^{th} = 24^{th} value = 61$

$Q_2 = X_{\frac{n}{2}+1}^{th} = 16^{th} value = 48$

New $Q_3 - Q_2 = 61 - 48 = 13$

$Q_2 - Q_1 = 48 - 37 = 11$

Since, $Q_3 - Q_2 > Q_2 - Q_1$ the distribution is positively skewed.

**Quartile coefficient of skewness**

$= \frac{(Q_3-Q_2)-(Q_2-Q_1)}{Q_3-Q_1}$ **or** $\frac{(Q_3+Q1)-(2MD)}{Q_3-Q_1}$

$= \frac{\mathbf{13-11}}{\mathbf{61-37}} = \mathbf{0.083}$

This indicates a positive skewness.

**ii)** Draw a frequency distribution table

| Length | Frequency ($f_i$) | Cumulative Frequency | Class point ($x_i$) | $x_i^2$ | $f_i * x_i$ | $f_i * x_i^2$ |
|---|---|---|---|---|---|---|
| 20 – 30 | 3 | 3 | 25 | 625 | 75 | 1875 |
| 30 – 40 | 7 | 10 | 35 | 1225 | 245 | 8575 |
| 40 – 50 | 8 | 18 | 45 | 2025 | 360 | 16200 |
| 50 – 60 | 5 | 23 | 55 | 3025 | 275 | 15125 |
| 60 – 70 | 4 | 27 | 65 | 4225 | 260 | 16900 |
| 70 – 80 | 3 | 30 | 75 | 5625 | 225 | 16875 |
| 80 – 90 | 1 | 31 | 85 | 7225 | 85 | 7225 |
| Total | 31 | | | | 1525 | 82775 |

$Mode = L_o + \frac{D_1}{D_1+D_2} * h$

Where: $L_o$: is the lower-class boundary of the model class

$D_1$: is the difference between frequency of model class and frequency of class before model class

$D_2$: is the difference between frequency of model class and frequency of class after model class

$h$: is the class width.

$L_o = 40, \quad D_1 = 8 - 7 = 1, \quad D_2 = 8 - 5 = 3, \quad and \quad h = 10$

$Mode = L_o + \frac{D_1}{D_1 + D_2} * h = 40 + \frac{1}{1+3} * 10 = 40 + \frac{10}{4} = 40 + 2.5$

$Mode = 42.5$

**iii)** $Standard\ deviation = \sqrt{\frac{\sum f_i * x_i^2}{\sum f_i} - \bar{x}^2}$

$\bar{x} = \frac{\sum f_i * x_i}{\sum f_i} = \frac{1525}{31} \cong 49.2$

$Standard\ deviation = \sqrt{\frac{\sum f_i * x_i^2}{\sum f_i} - \bar{x}^2} = \sqrt{\frac{82775}{31} - \left(\frac{1525}{31}\right)^2} \cong 15.82$

**iv)** $Coefficient\ of\ variation = \frac{\sigma}{\bar{x}} * 100$

$= \frac{15.82}{49.2} * 100 = 32.15\ \%$

**QUESTION TWO**

**Marking Guide**

| | criteria | Marks |
|---|---|---|
| **a. i** | Award 1 Mark for the correct answer of probability of success | **1** |
| | Award 1 Mark for the correct answer of probability of failure | **1** |
| | Award 1 mark for identification of the model to be used (independence a binomial model) | **1** |
| | Award 1 mark for formula of probability of success when x=2 | **1** |
| | Award 1 mark for correct answer of probability that 2 customers will pay with credit card | **1** |
| **a. ii** | Award 1 mark for formula of probability that more than 7 customers will pay by credit cards. | **1** |
| | Award 2 marks for correct answer of probability that more than 7 customers will pay with credit card | **2** |
| **a. iii** | Award 1 mark for the correct answer of the mean and standard deviation each | **2** |
| **Sub-total A** | | **10** |

| | | | |
|---|---|---|---|
| **b.** | **i** | Award 1 Mark for formulation of Null hypothesis Ho | **1** |
| | | Award 1 Mark for formulation of Alternative hypothesis $H_1$ | **1** |
| | | Award 1 mark for formula of probability that 11 patients have been positively impacted by the vaccine $P(X \leq 11)$ | **1** |
| | | Award 2 Marks for the correct answer from the test | **2** |
| | | Award 1 Mark for mentioning that boundary for the critical region will be to the left of $x = 11$ | **1** |
| | | Award 2 Marks for the correct decision (Ho will not be rejected) | **2** |
| **b.ii** | | Award 2 Marks for the correct reasons mentioned by the student | **2** |
| **Sub-total B** | | | **10** |
| **Total** | | | **20** |

**Model Answer**

**Solution A**

Let x be the number of the customers in a sample of ten customers who pay by credit card

Let paying by credit card be the probability of success, $P = 60\ \% = 0.6$

Let q be the probability of failure, 1-P=q

$q = 1 - 0.6 = 0.4$ be the probability of failure

Assuming independence a binomial $\beta$ model can be used with $n = 10$

Therefore, $X \sim \beta(10{,}6)$

i. $P(X = 2) = C_2^{10} * P^k * q^{n-k} = C_2^{10} * P^2 * q^8$

$= \frac{10!}{2!(10-2)!} * (0.6)^2 * (0.4)^8$

$= 45 * (0.6)^2 * (0.4)^8 = 0.011$

ii. $P(X > 7) = P(X = 8) + P(X = 9) + P(X = 10)$

$= C_8^{10} * P^8 * q^2 + C_9^{10} * P^9 q^1 + C_{10}^{10} * P^{10} q^0$

$= 45 * (0.6)^8 * (0.4)^2 + 10 * (0.6)^9 * (0.4)^1 + (0.6)^{10}$

$= 0.17$

**iii)** $\boldsymbol{The\ mean} = E(x) = n * p = 10 * (0.6) = 6$ and

**The standard deviation** $= \sqrt{\sigma^2} = \sqrt{n * p * q} = \sqrt{10 * (0.6) * (0.4)} = 1.55$

**Solution B**

i) Let x be the number of patients in 15 whose pandemic are relieved by the vaccine.

Assume that the effect of vaccine on patient is independent of the effect of the other patients

X can be modeled by a binomial distribution

Where, $X \sim \beta(15, p)$

$H_o$: $P = 0.9$ $(the\ success\ rate)$

$H_1$: $P < 0.9$

If $H_o$ is True, then, $X \sim \beta(15{,}0.9)$

Since the alternative hypothesis is $P < 0.9$,

The critical region is in the lower tail of the distribution. Therefore, let use a one-tailed test at 5 % level.

The test value, x, will lie in the critical region if $P(X \leq x) < 5\%$

Reject $H_o$ if $P(X \leq x) \leq 5\%$

Among 15 patients 11 react on their pandemic symptoms and 4 do not this means

$P(X \leq 11 \setminus P = 0.9) = P(X \geq 4 \setminus P = 0.1)$

$= 1 - P(X \leq 3)$

$= 1 - 0.9444$

$= 0.0556 \cong 5.6\ \%$

Or

$P(X \leq 11) = 1 - P(X \geq 12)$

$= 1 - \left[C_{12}^{15} * (0.9)^{12} * (0.1)^{3} + C_{13}^{15} * (0.9)^{13} * (0.1)^{2} + C_{14}^{15} * (0.9)^{14} * (0.1)^{1} + C_{15}^{15} * (0.9)^{15}\right]$

$= 1 - 0.944 = 0.0556 \cong 5.6\ \%$

$P(X \leq 11)$ is greater than 5 %. This means that boundary for the critical region will be to the left of $x = 11$.

Therefore, Ho is not rejected and the vaccine company's claim of a 90 % success rate is upheld.

ii) With safety in mind, it would be wise to suggest that the doctor errors on the side of caution of vaccine and carry out further tests before accepting that the success rate is 90 %, since this can have a direct effect on life.

**QUESTION THREE**

**Marking Guide**

| | criteria | Marks |
|---|---|---|
| **a. i** | Award 1 Mark for the correct meaning of population | **1** |
| **a. ii** | Award 1 Mark for the correct meaning of sampling frame | **1** |
| **b.** | Award 1 Mark for correct answer on each case | **2** |
| **c.** | Award 1 Mark for mentioning that every $k^{th}$ member should be chosen | **1** |
| | Award 1 Mark for formula of how to obtain K value | **1** |
| | Award 1 Mark for example of 8 members selected from a list of 300 | **1** |
| **d.i** | Award 1 Mark for each correct line in Leontief matrix | **3** |
| **d.ii** | Ward 1 Mark for the correct answer of $\det(I - A)$ | **1** |
| | Award 3 Marks for calculation of output 1 ($x_1$) | **3** |
| | Award 3 Marks for calculation of output 2 ($x_2$) | **3** |
| | Award 3 Marks for calculation of output 3 ($x_3$) | **3** |

**Model Answer**

**Solution A**

**i)** A population is a particular group of individuals or items.

**ii)** One of individual members of a population have been numbered to form a list, this list is called a sampling frame.

**Solution B**

The suitable sampling frame for the case One is the list of registered owners as kept by Yego Carbs cooperative in Kigali city. Whereas the suitable sampling frame for the case Two is a list compiled from information provided by the Ministry of Health/RBC.

**Solution C**

To choose a systematic sample of 8 members from a list of 300 you can proceed as follow:

- ✓ Since you are going to choose every $k^{th}$ member, you need to find a suitable value of k.
- ✓ To choose so, choose a convenient value close to $\frac{N}{n}$.
- ✓ Where $N$ is a population size and $n$ is the sample size.
- ✓ In this case $\frac{N}{n} = \frac{300}{8} = 37.5, so\ k = 40\ will\ do.$
- ✓ Then we choose a random starting point

For example: Let Ran# be a random selection. If Ran# in your calculator, it gives you 0.870

Take the first member of the sample as 87 and then add 40 each time. The other member is 127, 167, 207, 247, 287, 27, and 67. Then you can arrange

**Solution D**

Given A = $\begin{pmatrix} 0.3 & 0.5 & 0.2 \\ 0.2 & 0.0 & 0.5 \\ 0.1 & 0.3 & 0.1 \end{pmatrix}$

i) The Leontief matrix = $I - A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0.3 & 0.5 & 0.2 \\ 0.2 & 0 & 0.5 \\ 0.1 & 0.3 & 0.1 \end{pmatrix} = \begin{pmatrix} 0.7 & -0.5 & -0.2 \\ -0.2 & 1 & -0.5 \\ -0.1 & -0.3 & 0.9 \end{pmatrix}$

ii) Let $X^T = (x_1, x_2, x_3) = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

Now $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = (I - A)^{-1} * D$

$$= \begin{pmatrix} 0.7 & -0.5 & -0.2 \\ -0.2 & 1 & -0.5 \\ -0.1 & -0.3 & 0.9 \end{pmatrix}^{-1} * \begin{pmatrix} 100 \\ 40 \\ 50 \end{pmatrix}$$

$(I - A)^{-1} = \frac{1}{\det(I-A)} * cofactor\ matrix$

$\det(I - A) = 0.401$

Therefore, $\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \frac{1}{0.401} * \begin{pmatrix} 0.75 & 0.42 & 0.40 \\ 0.23 & 0.61 & 0.39 \\ 0.16 & 0.25 & 0.62 \end{pmatrix} * \begin{pmatrix} 100 \\ 40 \\ 50 \end{pmatrix}$

$x1 = \frac{1}{0.401} * [0.75 * (100) + 0.42 * (40) + 0.40 * (50)]$

$x1 = 270$

$x2 = \frac{1}{0.401} * [0.23 * (100) + 0.61 * (40) + 0.39 * (50)]$

$x2 = 167$

$x3 = \frac{1}{0.401} * [0.16 * (100) + 0.25 * (40) + 0.62 * (50)]$

$x3 = 147$

Therefore, the output matrix is:

$\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} 270 \\ 167 \\ 147 \end{pmatrix}$

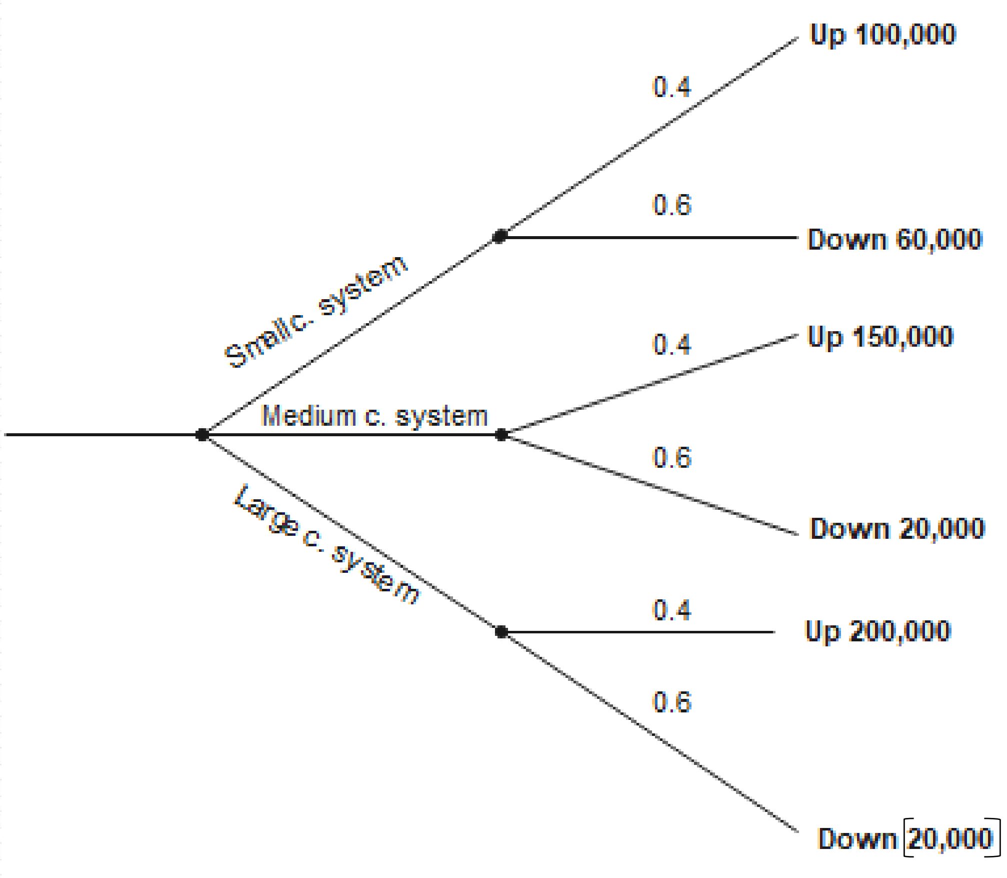
## QUESTION FOUR

| **Marking Guide** | **Marks** |
|---|---|
| **a)** 1 Mark for each correct alternative on the tree Maximum 3 | 3 |
| 1 Mark for calculation of EMV on each alternative Maximum | 3 |
| 1 Marks for correct decision | 1 |
| **Subtotal** | **7** |
| b. i) Develop objective function | 1 |
| Develop constraint function (1 for demand constraint and 1 for supply constraint) | 2 |
| **Maximum** | **3** |
| **b. ii)** 0.5 Marks for each Correct allocation using NWC (Maximum 2.5) | 2.5 |
| 1.5 Marks for calculation of total cost using NWC | 1.5 |
| 0.5 Marks for each Correct allocation using LCM (Maximum 2.5) | 2.5 |
| 1.5 Marks for calculation of total cost using LCM | 1.5 |
| 0.5 Marks for correct decision | 0.5 |
| **Subtotal** | **8.5** |
| **b.iii)** 0.5 Marks for each characteristic of the transportation problem | 1.5 |
| **Total** | **20** |

**Model Answer**

**Solution A**

The tree diagram is as follow:




EMV(s) = 100,000*(0.4) + 60,000*(0.6) = 40,000 + 36,000 = $ 76,000

EMV(m) = 150,000 *(0.4) + 20,000*(0.6) = 60,000 + 12,000 = $ 72,000

EMV(l) = 200,000*(0.4) – 20,000*(0.6) = 80,000 – 12,000 = $ 68,000

Using expected approach, the company may use a small computer system to have the maximum profit.

**Solution B**

i) mathematical model

Let $x_{ij}$ be the number of the units kg transported from supply I to demand J.

Therefore, $Minimize\ z = 6x_{11} + 6x_{12} + 6x_{13} + 4x_{14} + 4x_{21} + 9x_{22} + 4x_{23} + 5x_{24} + 5x_{31} + 6x_{32} + 7x_{33} + 8x_{34}$

Subject to:

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 1000$$
$$x_{21} + x_{22} + x_{23} + x_{24} \leq 700$$
$$x_{31} + x_{32} + x_{33} + x_{34} \leq 900$$
Supply constraints

$$x_{11} + x_{21} + x_{31} \geq 900$$
$$x_{12} + x_{22} + x_{32} \geq 800$$
$$x_{13} + x_{23} + x_{33} \geq 500$$
$$x_{14} + x_{24} + x_{34} \geq 400$$
Demand constraints

ii) The information can be summarized in the table below:

| | A | B | C | D | Supply |
|---|---|---|---|---|---|
| X | 6 | 6 | 6 | 4 | 1000 |
| Y | 4 | 2 | 4 | 5 | 700 |
| Z | 5 | 6 | 7 | 8 | 900 |
| Demand | 900 | 800 | 500 | 400 | 2600 |

**North West Corner Method (NWC)**

| Destination/sources | A | B | C | D | Supply |
|---|---|---|---|---|---|
| X | 6 900 | 6 100 | 6 | 4 | ~~1000~~ ~~100~~ 0 |
| Y | 4 | 2 700 | 4 | 5 | ~~700~~ 0 |
| Z | 5 | 6 | 7 500 | 8 400 | ~~900~~ ~~400~~ |
| Demand | ~~900~~ 0 | ~~800~~ ~~700~~ | ~~500~~ 0 | ~~400~~ | 2600 |

$x_{11} = 900$

$x_{12} = 100$

$x_{22} = 700$

$x_{33} = 500$

$x_{34} = 400$

The optimum is $Z = 6 * (900) + 6 * (100) + 2 * (700) + 7 * (500) + 8 * (400)$

$Z = 5400 + 600 + 1400 + 3500 + 3200$

$Z = 14{,}100$

**Using Least Cost Methods (LCM)**

| Destination/Sources | A | B | C | D | Supply |
|---|---|---|---|---|---|
| X | 6 | 6 100 | 6 500 | 4 400 | 1000 600 0 |
| Y | 4 | 2…….700 | 4 | 5 | 700 0 |
| Z | 5 900 | 6 | 7 | 8 | 900 0 |
| Demand | 900 0 | 800 100 0 | 500 0 | 400 0 | |

$x_{12} = 100$

$x_{13} = 500$

$x_{14} = 400$

$x_{22} = 700$

$x_{31} = 900$

The optimum is $Z = 6 * (100) + 6 * (500) + 4 * (400) + 2 * (700) + 5 * (900)$

$Z = 600 + 3000 + 1600 + 1400 + 4500$

$Z = 11{,}100$

**The best method is Least cost method since it gives the minimum cost.**

iii) A transportation problem is characterized by three elements:

1. Supply points

2. Demand points

3. Cost of transport

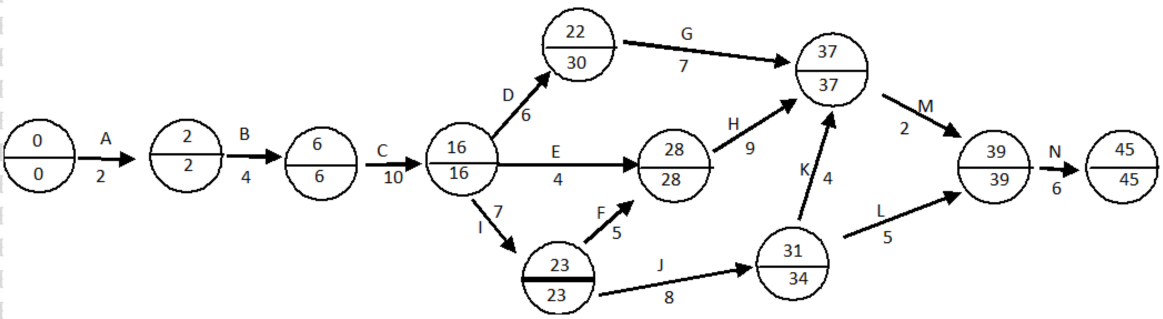
**QUESTION FIVE**

| **Marking Guide** | **Marks** |
|---|---|
| Draw network (0.5Marks for each correct drawn activity, maximum 7) | 7 |
| Identify critical (0.5 for each activity on CP) | 4 |
| Calculation of mean and the variance (2 Mark for mean and 3 marks for variance) | 5 |
| Explanation for the concept of crashing of a project | 2 |
| Explanation for the criteria for selecting of an activity for crashing (0.5 for each criteria) | 2 |
| **Total** | **20** |

**Model Answer**

**Solution A**

i) the project Network




ii) Using the diagram, the critical path is $A - B - C - I - F - H - M - N$

iii) The mean and the variance of the project

| **Activity** | $t_O$ | $t_M$ | $t_P$ | **Mean or** $t_e = \frac{t_O + 4t_M + t_P}{6}$ | **Variance or** $t_o = \left(\frac{t_P - t_o}{6}\right)^2$ |
|---|---|---|---|---|---|
| A | 1 | 2 | 3 | 2 | $\frac{1}{9}$ |
| B | 2 | 3.5 | 8 | 4 | 1 |
| C | 6 | 9 | 18 | 10 | 4 |
| D | 4 | 5.5 | 10 | 6 | 1 |
| E | 1 | 4.5 | 5 | 4 | $\frac{4}{9}$ |
| F | 4 | 4 | 10 | 5 | 1 |
| G | 5 | 6.5 | 11 | 7 | 1 |
| H | 5 | 8 | 17 | 9 | 4 |
| I | 3 | 7.5 | 9 | 7 | 1 |
| J | 3 | 9 | 9 | 8 | 1 |
| K | 4 | 4 | 4 | 4 | 0 |
| L | 1 | 5.5 | 7 | 5 | 1 |
| M | 1 | 2 | 1 | 2 | 3 |
| N | 5 | 5.5 | 9 | 6 | $\frac{4}{9}$ |

The mean $= 2 + 4 + 10 + 7 + 5 + 9 + 2 + 6 = 45$

The variance $\sigma^2 = \frac{1}{9} + 1 + 4 + 1 + 1 + 4 + 3 + \frac{4}{9} = 14.5556$

iv) The process of shortening the time to complete a project is called crashing and is usually achieved by putting into service additional labor or machines to one activity or more activities. Crashing involves more costs to speed up a project by spending extra cost as possible. Project crashing seeks to minimize the extra cost for completion of a project before the stipulated time.

v) The criteria for the selection of an activity to be crashed is as follows:

select the activity on the critical path with smallest crash cost per unit time. Crash this activity to the maximum units of time as may be permissible by the given data.

Crashing an activity requires extra amount to be spent. We have to select an activity with less crash cost.

**QUESTION SIX**

| **Marking Guide** | **Marks** |
|---|---|
| a.i)Calculation of Mean x | 1 |
| Calculation of Mean y | 1 |
| Calculation of sum of xy ($\sum$xy) | 1 |
| Calculation of sum of y squared ($y^2$) | 1 |
| Calculation of b (d) (regression coefficient of y on x) | 2 |
| Calculation of c (a) (constant) | 2 |
| Regression line equation | 1 |
| **a.ii)** 2 Marks for the calculation of methew's age | 2 |
| b. i) 3 Mark for maximin row and minimax column | 3 |
| b.ii) 1 Mark for the 3x3 reduced matrix, | 1 |
| 1 Mark for the 3x2 reduced matrix, | 1 |
| 1 Mark for the 2x2 reduced matrix, | 1 |
| b.iii) 1 mark for p and 1-p, | 1 |
| 1 Mark for r and 1-q, | 1 |
| 1 Mark for v | 1 |
| **Total** | **20** |

**Model Answer**

**Solution A**

i) The least-square line of regression of X on Y: This equation is used to estimate a value of X for a given value of Y.

$$X = c + dY \text{ where } c = \bar{x} - d\bar{y} \text{ and } d = \frac{s_{xy}}{s_{yy}}$$

$$\bar{x} = \frac{\sum x}{n} = \frac{327}{6} \text{ and } \bar{y} = \frac{\sum y}{n} = \frac{1661}{6}$$

| | X | Y | x^2 | y^2 | xy |
|---|---|---|---|---|---|
| 1 | 42 | 143 | 1764 | 20449 | 6006 |
| 2 | 50 | 179 | 2500 | 32041 | 8950 |
| 3 | 47 | 197 | 2209 | 38809 | 9259 |
| 4 | 58 | 335 | 3364 | 112225 | 19430 |
| 5 | 57 | 384 | 3249 | 147456 | 21888 |
| 6 | 73 | 423 | 5329 | 178929 | 30879 |
| Total | 327 | 1661 | 18415 | 529909 | 96412 |

$$s_{xy} = \frac{1}{n}\sum xy - \bar{x}\bar{y} = \frac{1}{6} * 96412 - \frac{327}{6} * \frac{1661}{6}$$

$$s_{xy} = 981.25$$

$$s_{yy} = \frac{1}{n}\sum y^2 - \bar{y}^2 = \frac{1}{6} * 529909 - \left(\frac{1661}{6}\right)^2 = 11681.47$$

$$d = \frac{s_{xy}}{s_{yy}} = \frac{981.25}{11681.47} = 0.084$$

Calculate c

$$c = \bar{x} - d\bar{y} = \frac{327}{6} - 0.084 * \frac{1661}{6} = 31.24$$

**The equation of regression line of x on y is $x = 31.2 + 0.084y$**

**Note: This alternative approach can be used to find regression line equation**

$b_{xy}$= Slope of the line of regression of X on Y and given by

$$b_{XY} = \frac{n\sum XY - (\sum X)(\sum Y)}{n\sum Y^2 - (\sum Y)^2}$$

The line of regression of X on Y passes through the point $(\bar{x}\bar{y})$ and hence the equation of the line of regression of X on Y (X = c + dY) can also be written as

$$X - \bar{X} = b_{XY}(Y - \bar{Y})$$

ii) From the equation, when $y = 582$, $x = 31.2 + 0.084 * 582 = 80$
Therefore Mathew is 80 years old

**Solution B**

i) Let compute the min for each row and the max for each column

| Strategy | | Player 2 | | | | Min |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | |
| Player 1 | 1 | 0 | -2 | 2 | 1 | -2 |
| | 2 | 5 | 4 | -3 | 5 | -3 |
| | 3 | 2 | 3 | -4 | 3 | -4 |
| Max | | 5 | 4 | 2 | 5 | |

**There is no saddle point.**

ii) Using dominated strategies technique to eliminate rows and / or column we get:

| Strategy | | Player 2 | |
|---|---|---|---|
| | | III | VI |
| Player 1 | A | 2 | 1 |
| | B | -4 | 3 |

iii) Algebraic methods

$P = \frac{3-(-4)}{8} = \frac{7}{8}$ $and$ $1 - P = \frac{1}{8}$

$q = \frac{3-1}{8} = \frac{2}{8}$ $and$ $1 - q = \frac{6}{8}$

$V = \frac{6+4}{8} = \frac{10}{8} = \frac{5}{4}$

**Therefore, the value of the game is $\frac{5}{4}$**

## QUESTION SEVEN.

| Marking Guide | Marks |
|---|---|
| a.i) 0.5 Marks for each forecasted number (Maximum 2) | 2 |
| a.ii) 0.5 Marks for each forecasted number ( Maximum 2.5) | 2.5 |
| a.iii) 1 Mark for mean squared deviation of moving average | 1 |
| 1 Mark for mean squared deviation of exponential smoothing | 1 |
| b. i) 0.5 Mark for formula of each index (Maximum 1.5 Marks) | 1.5 |
| 2 Marks for calculation of each index (Maximum 6 Marks) | 6 |
| b.ii) 0.5 for each desirable properties of the base period in index number (Maximum 2) | 2 |
| 1 for each explained desirable properties of the base period in index number (Max 4) | 4 |
| **Total** | **20** |

**Model Answer**

**Solution A**

i) The two months moving average $m_i$ for month two to five (I vary from 2 to 5) is given by:

$$m_2 = \frac{13+17}{2} = 15$$

$$m_3 = \frac{17+19}{2} = 18$$

$$m_4 = \frac{19+23}{2} = 21$$

$$m_5 = \frac{23+24}{2} = 23.5$$

The forecast for month six is just the moving average for the month before that. i.e. the moving average for month 5 =$m_5 = FRW23{,}500$

ii) Applying exponential smoothing with a smooth constant 0.9 we get:

$$m_1 = y_1 = 13$$

$$m_2 = 0.9 * y_2 + 0.1 * m_1 = (0.9 * 17) + (0.1 * 13) = 16.60$$

$$m_3 = 0.9 * y_3 + 0.1 * m_2 = (0.9 * 19) + (0.1 * 16.60) = 18.76$$

$$m_4 = 0.9 * y_3 + 0.1 * m_4 = (0.9 * 23) + (0.1 * 18.76) = 22.58$$

$$m_5 = 0.9 * y_5 + 0.1 * m_4 = (0.9 * 24) + (0.1 * 22.58) = 23.86$$

As before the forecast for month six is just the average for month 5

$$m_5 = FRW23{,}860$$

iii) To compare the two forecasts, we have to calculate the mean squared deviation (MSD)

$$MSD = \frac{[(15-19)^2+(18-23)^2+(21-24)^2]}{3} = 16.67$$

For the exponential,

$$MSD = \frac{[(13-17)^2+(16.60-19)^2+(18.76-23)^2+(22.58-24)^2]}{4}$$

$$MSD = 10.44$$

Since the exponential smoothing appears to give the best one month ahead as it has a lower MSD, we prefer the forecast of FRWFR23,860 produced by exponential smoothing.

**Solution B**

**i)** Before calculation of the indexes, let us start from the table below:

| Commodity | Base | | Current | | $p_0q_0$ | $P_1q_0$ | $P_0q_1$ | $P_1q_1$ |
|---|---|---|---|---|---|---|---|---|
| | $P_0$ | $q_0$ | $P_1$ | $Q_1$ | | | | |
| Irish potatoes kinigi | 450 | 5,000 | 500 | 5,400 | 2,250,000 | 2,500,000 | 2,430,000 | 2,700,000 |
| casava | 260 | 1,000 | 300 | 1,100 | 260,000 | 300,000 | 286,000 | 330,000 |
| Sweet potato | 230 | 15,000 | 340 | 13,000 | 3,450,000 | 5,100,000 | 2,990,000 | 4,420,000 |
| yam | 310 | 500 | 320 | 500 | 155,000 | 160,000 | 155,000 | 160,000 |
| Matoke | 200 | 7,000 | 260 | 6,800 | 1,400,000 | 1,820,000 | 1,360,000 | 1,768,000 |
| | | | | | 7,515,000 | 9,880,000 | 7,221,000 | 9,378,000 |

Laspeyres Formula In/0= $\frac{\sum P1 \cdot Q0}{\sum P0 \cdot Q0}$*100

Therefore, Laspeyres index $= \frac{9,880,000}{7,515,000}$*100 = 131.47

Paasche Formula In/0= $\frac{\sum P1 \cdot Q1}{\sum P0 \cdot Q1}$*100

Therefore, Paasche index $\frac{9,378,000}{7,221,000}$*100=129.9

Fisher Ideal Formula = $\sqrt{\frac{\sum P1\cdot Q0}{\sum P0\cdot Q0} * \frac{\sum P1\cdot Q1}{\sum P0\cdot Q1}}$ *100

Therefore, Fisher ideal index = $\sqrt{\frac{9,880,000}{7,515,000} * \frac{9,378,000}{7,221,000}}$ *100= 130.7

**ii)** Desirable properties of the base period in index number are:

1. **The base year should not be either too short or too long:** It should not be either less than a month or more than a year for calculation purpose

**2.** **The base year should not belong to too near or too far:** This means that comparison of current year's conditions with the conditions in the base year. It means if the base year is too near to the current year, then comparison fail to capture the changes. Thus, in order to conduct a meaningful comparison, the base year should not be either too far or to near to the current year

**3.** **The base year should be so selected that the data for the same should be available:** The data for a year should be available in order to regard that particular year to be the base year. This enables one to draw conclusions, inferences and for making comparisons

**4.** **The base year period should be constantly updated:** The base year should be constantly updated due to the changes in taste, preferences and fashion otherwise; the comparison becomes misleading or inconclusive

**END OF MARKING GUIDE AND MODEL ANSWERS**